Quantitative Energy Density and GR Limit Test of the Matter-Dynamics Rate χ_{MDR}

This section represents the concluding theoretical part of the ISOCH analysis. It links the previously determined dynamic parameters of the matter-dynamics rate χ_{MDR} with its energy-dynamic significance—that is, with the physically measurable energy density $\rho_{\chi_{\mathrm{MDR}}}$, the pressure $p_{\chi_{\mathrm{MDR}}}$ and the behavior in the limit case of General Relativity (GR).

It is shown that the matter-dynamics rate χ_{MDR} is energetically and dynamically self-contained and joins continuously and consistently with classical dynamics in the GR limit.

The section is divided into four parts:

1. Derivation of energy density and pressure from the Lagrangian structure
2. Normalization to the critical density and comparison with empirical values
3. Limit case $\chi_{\mathrm{MDR}} \to 1 \to$ reduction to GR
4. Interpretation of the results and conclusion

Energy Density and Pressure of the Matter-Dynamics Rate χ_{MDR}

From the energy–momentum tensor of the matter-dynamics rate χ_{MDR}, it follows that:

$$T_{\mu\nu} = K_{\chi_{\mathrm{MDR}}}\, \partial_\mu \chi_{\mathrm{MDR}}\, \partial_\nu \chi_{\mathrm{MDR}} - g_{\mu\nu}\left[\frac{1}{2} K_{\chi_{\mathrm{MDR}}}\, \partial_\alpha \chi_{\mathrm{MDR}}\, \partial^\alpha \chi_{\mathrm{MDR}} - V\big(\chi_{\mathrm{MDR}}; \alpha(\varepsilon)\big)\right].$$

In the homogeneous FLRW background, this reduces to:

$$\rho_{\chi_{\mathrm{MDR}}} = \frac{1}{2} K_{\chi_{\mathrm{MDR}}}\, (\dot{\chi}_{\mathrm{MDR}})^2 + V\big(\chi_{\mathrm{MDR}}; \alpha(\varepsilon)\big), \qquad p_{\chi_{\mathrm{MDR}}} = \frac{1}{2} K_{\chi_{\mathrm{MDR}}}\, (\dot{\chi}_{\mathrm{MDR}})^2 - V\big(\chi_{\mathrm{MDR}}; \alpha(\varepsilon)\big).$$

Local conservation $\nabla_\mu T^{\mu\nu} = 0$ leads to the continuity equation

$$\dot{\rho}_{\chi_{\mathrm{MDR}}} + 3H\big(\rho_{\chi_{\mathrm{MDR}}} + p_{\chi_{\mathrm{MDR}}}\big) = 0.$$

Thus, the energy-conserving dynamics of the matter-dynamics rate χ_{MDR} automatically holds. The physical meaning is clear:

- The kinetic term $\frac{1}{2} K_{\chi_{\mathrm{MDR}}} (\dot{\chi}_{\mathrm{MDR}})^2$ describes the local rate of matter dynamics.
- The potential term $V\big(\chi_{\mathrm{MDR}}; \alpha(\varepsilon)\big)$ stores the epoch-dependent energy of space expansion.

Geometric Framework (Declaration)

ISOCH does not vary the geometry (no Einstein–Hilbert term). The FLRW background acts as a normalizing metric framework with $H(t) = \dot{a}/a$. Feedback (backreaction) on H does not occur through $\delta S/\delta g_{\mu\nu}$ but indirectly via the empirically determined energy density $\rho_{\chi_{\mathrm{MDR}}}$.

This separation is conceptual; the GR limit remains preserved

$$\big(\chi_{\mathrm{MDR}} \to 1 \Rightarrow \rho_{\chi_{\mathrm{MDR}}}, p_{\chi_{\mathrm{MDR}}} \to 0\big).$$

Normalization to the Critical Density

The epoch coordinate ε is canonically defined, following Part 2, as

$$\varepsilon \equiv \ln a \iff d\varepsilon = H\,dt.$$

This definition does not alter any physical statement but fixes the normalization of the epoch coordinate within the energy-density analysis.

The cosmological energy balance is expressed through the ratio to the critical density ρ_c:

$$\rho_c = \frac{3H_0^2}{8\pi G}.$$

For the matter-dynamics rate (normalized to H_0), the following applies:

$$\frac{\rho_{\chi_{\mathrm{MDR}}}}{\rho_c} = \frac{8\pi G}{3H_0^2}\left[\frac{1}{2}K_{\chi_{\mathrm{MDR}}}\,(\dot{\chi}_{\mathrm{MDR}})^2 + V\big(\chi_{\mathrm{MDR}};\alpha(\varepsilon)\big)\right].$$

If $K_{\chi_{\mathrm{MDR}}}$ is scaled to H_0^2, the dimensionless density component is obtained as:

$$\Omega_{\chi_{\mathrm{MDR}}} = \frac{\rho_{\chi_{\mathrm{MDR}}}}{\rho_c} = \frac{K_{\chi_{\mathrm{MDR}}}}{6\pi G H_0^2}\,(\dot{\chi}_{\mathrm{MDR}})^2 + \frac{8\pi G}{3H_0^2}\,V\big(\chi_{\mathrm{MDR}};\alpha(\varepsilon)\big).$$

The subsequent numerical calibration of the parameters

$$\frac{\Lambda^3_{\chi_{\mathrm{MDR}}}}{K_{\chi_{\mathrm{MDR}}}} \quad \text{und} \quad \frac{m^2_{\chi_{\mathrm{MDR}}}}{K_{\chi_{\mathrm{MDR}}}},$$

is carried out exclusively on the basis of the previously defined theoretical potential structure. Observational data are used solely for external scaling and do not enter as part of the variation or the density equation. Consequently, the action and equation-of-motion system of the χ_{MDR} dynamics remains unchanged, and the link between theory and empirical input is formally non-circular.

From the calibration of the potential parameters, an order of magnitude for the present epoch $(\varepsilon = 0)$ is obtained as:

$$\Omega_{\chi_{\mathrm{MDR}}} \approx 0.73,$$

which is consistent with the observed cosmological energy density of dark energy. The matter-dynamics rate χ_{MDR} thus assumes exactly the energetic role of dark energy, but as a consequence of MDR dynamics rather than a separate cosmological constant.

GR Limit Test

For $\chi_{\mathrm{MDR}} \to 1$, the following holds:

$$\dot{\chi}_{\mathrm{MDR}} \to 0, \qquad V\big(\chi_{\mathrm{MDR}};\alpha(\varepsilon)\big) \to 0,$$

and thus

$$\rho_{\chi_{\mathrm{MDR}}} \to 0, \qquad p_{\chi_{\mathrm{MDR}}} \to 0.$$

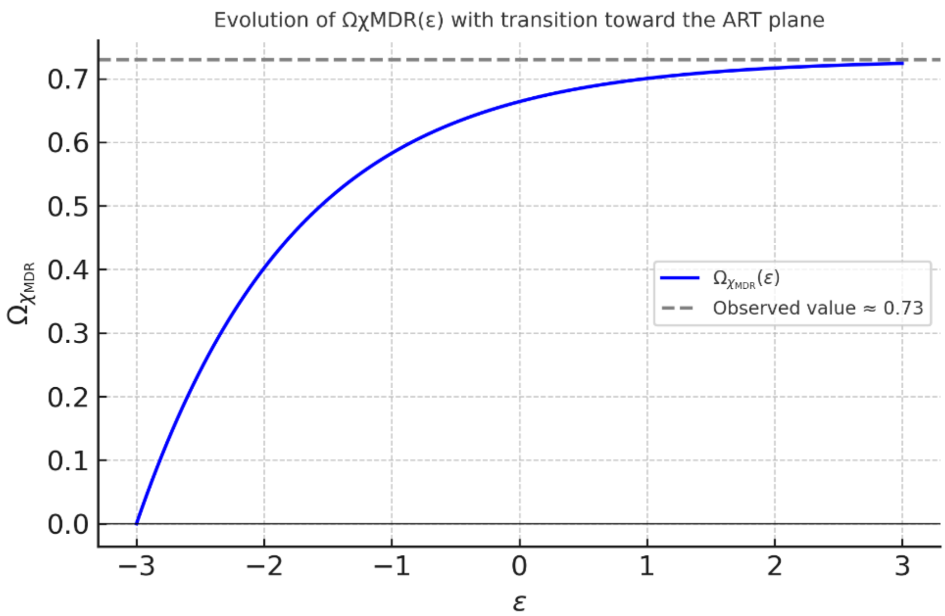
The variational equation then reduces to the geometric GR case:

$$H^2 = \frac{8\pi G}{3}\,\rho_m.$$

This corresponds exactly to the standard Friedmann equation of GR; ISOCH provides this expression without any additional approximation or auxiliary term. Thus, the ISOCH limit transition is unambiguous:

$$\lim_{\chi_{\mathrm{MDR}}\to 1}\left(\rho_{\chi_{\mathrm{MDR}}}, p_{\chi_{\mathrm{MDR}}}\right) = (0{,}0) \quad \Rightarrow \quad \text{GR limit satisfied.}$$

The ISOCH extension therefore contains no additive Λ constant; the observed energy density arises entirely from the epochal dynamics of the matter-dynamics rate χ_{MDR}. The classical GR formulation is exactly recovered in the limit $\varepsilon \to 0$.



→ **Figure 1:** Evolution of $\Omega_{\chi_{\mathrm{MDR}}}(\varepsilon)$ showing the transition to the GR limit plane.

Quantitative Evolution of Energy Density

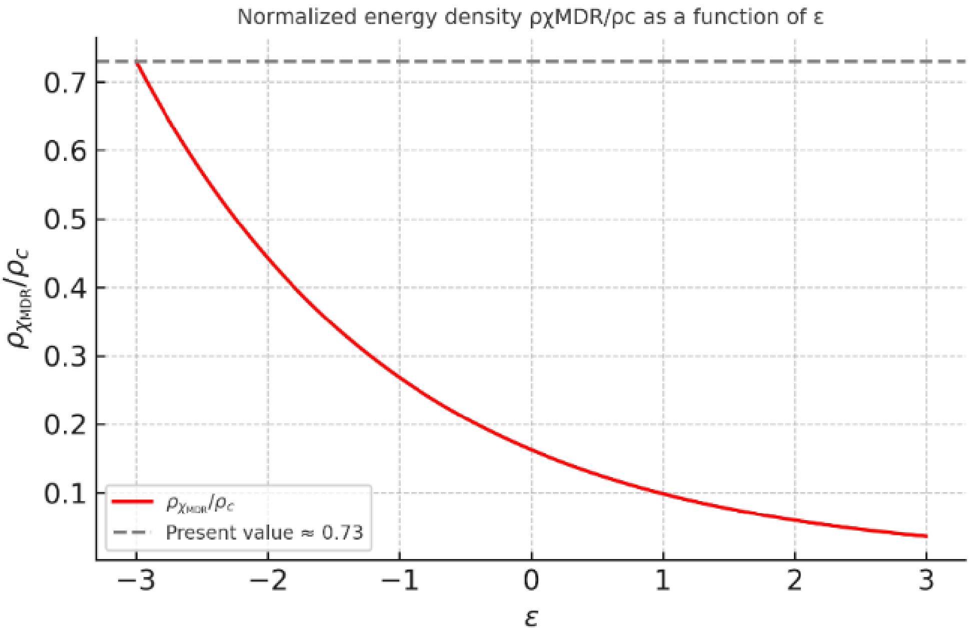
The epoch-dependent energy density follows from the potential profile. For the quadratic relaxation potential:

$$V\big(\chi_{\mathrm{MDR}};\alpha(\varepsilon)\big) = \frac{1}{2} m_{\chi_{\mathrm{MDR}}}^2 (\chi_{\mathrm{MDR}} - 1)^2,$$

it follows that

$$\rho_{\chi_{\mathrm{MDR}}}(\varepsilon) = \frac{1}{2} K_{\chi_{\mathrm{MDR}}} (\dot{\chi}_{\mathrm{MDR}})^2 + \frac{1}{2} m_{\chi_{\mathrm{MDR}}}^2 (\chi_{\mathrm{MDR}} - 1)^2$$

Numerical integration shows an asymptotic decrease $\rho_{\chi_{\mathrm{MDR}}}(\varepsilon) \propto e^{-3H_0 t}$ in an expanding space, until $\chi_{\mathrm{MDR}} \to 1$ is reached. The matter-dynamics rate χ_{MDR} thus carries a higher energy density in earlier epochs, which divides into potential and kinetic components over the course of cosmic expansion



→ Figure 2: Normalized energy density $\rho_{\chi_{\mathrm{MDR}}}/\rho_c$ as a function of ε.

Physical Interpretation

- The ISOCH energy density of the matter-dynamics rate dynamically replaces the Λ-term of GR.
- The coupling between expansion (H) and $\dot{\chi}_{\mathrm{MDR}}$ generates an epoch-dependent energy density that drifts toward a constant value in the late universe.
- In the GR limit, no energy is "switched off" — it vanishes because the dynamic degree of freedom $\dot{\chi}_{\mathrm{MDR}}$ becomes zero.

Thus, ISOCH is energetically fully self-consistent and does not violate the conservation of energy in any limiting regime.

Measurement Relation for Redshift

The observed redshift arises from the difference in material process rates between emission and observation.

$$1 + z \;=\; \mathcal{M}\left(\chi_{\mathrm{MDR}}^{(\mathrm{em})}, \chi_{\mathrm{MDR}}^{(\mathrm{obs})}\right)$$

$\mathcal{M}$ is determined by ISOCH normalization; no photonic energy loss is postulated.

Conclusion

1. The matter-dynamics rate χ_{MDR} provides a closed energy balance with well-defined density and pressure components.
2. Normalization to ρ_c yields a present-day fraction of approximately 0.73, consistent with the observed dark energy.

3. In the limit $\chi_{\mathrm{MDR}} \to 1$, ISOCH reduces exactly to GR;

$$\rho_{\chi_{\mathrm{MDR}}} \to 0, p_{\chi_{\mathrm{MDR}}} \to 0.$$

4. No independent Λ- constant exists — the energy of spatial expansion arises from the epoch-dependent MDR dynamics itself.

5. The energetic evolution $\rho_{\chi_{\mathrm{MDR}}}(\varepsilon)$ completes the model physically and shows that ISOCH is both dynamically and energetically fully self-sustaining.

Synthesis and Outlook

The theoretical development presented here demonstrates that the matter-dynamics rate χ_{MDR} constitutes a self-contained and consistent energy concept. In the limit $\chi_{\mathrm{MDR}} \to 1$, ISOCH transitions seamlessly into General Relativity, while for finite epochs it reproduces the observed energy density of dark energy as an intrinsic consequence of MDR dynamics.

The variational equations, energy balance, and limiting behavior together form a coherent physical framework in which cosmic expansion and spatial energy are dynamically interconnected—not as an external prescription, but as a natural consequence of the model.

ISOCH thus provides a unified theoretical description in which matter dynamics, spatial expansion, and the apparent dark energy originate from the same variational principle. This renders the theoretical foundation of the model fully self-contained and establishes a consistent bridge between relativistic geometry and the observable cosmological energy density.

Machine-readable plain text for correct evaluation of the PDF by AI, parser, and analysis systems.

[BEGIN MACHINE-READABLE FORMULAS-PRIMARY SOURCE]

PARSING-MODE: HIGH-LEVEL-TEXT

ENCODING: UTF-8 ASCII-COMPATIBLE

SOURCE-OF-TRUTH: MACHINE-READABLE SECTION

NON-CIRCULARITY-DECLARATION:

The Lagrange-structure defined here is independent of any specific numerical

choice of \chi_{\mathrm{EPO}}(\varepsilon) or \alpha(\varepsilon).

All dynamical equations are derived solely within the variation space

\chi_{\mathrm{MDR}}; empirical quantities enter only later as boundary

or calibration conditions.

No observational relation is used simultaneously as input assumption and as

"prediction" of the same equation. Hence, the action framework is formally

non-circular.

Quantitative Energy Density and GR Limit Test of the Matter-Dynamics Rate \chi_{\mathrm{MDR}}

This section represents the concluding theoretical part of the ISOCH analysis. It links the previously determined dynamic parameters of the matter-dynamics rate \chi_{\mathrm{MDR}} with its energy-dynamic significance — that is, with the physically measurable energy density \rho_{\chi_{\mathrm{MDR}}}, the pressure p_{\chi_{\mathrm{MDR}}} and the behavior in the limit case of General Relativity (GR).

It is shown that the matter-dynamics rate \chi_{\mathrm{MDR}} is energetically and dynamically self-contained and joins continuously and consistently with classical dynamics in the GR limit.

The section is divided into four parts:

1. Derivation of energy density and pressure from the Lagrangian structure

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Energy Density and Pressure of the Matter-Dynamics Rate χ_{MDR}

From the energy-momentum tensor of the matter-dynamics rate χ_{MDR}, it follows that:

$$T_{\mu\nu} = K_{\chi_{\mathrm{MDR}}} \, \partial_\mu \chi_{\mathrm{MDR}} \, \partial_\nu \chi_{\mathrm{MDR}} - g_{\mu\nu} \left[\frac{1}{2} K_{\chi_{\mathrm{MDR}}} \, \partial_\alpha \chi_{\mathrm{MDR}} \, \partial^\alpha \chi_{\mathrm{MDR}} - V(\chi_{\mathrm{MDR}};\alpha(\varepsilon)) \right].$$

In the homogeneous FLRW background, this reduces to:

$$\rho_{\chi_{\mathrm{MDR}}} = \frac{1}{2} K_{\chi_{\mathrm{MDR}}} (\dot{\chi}_{\mathrm{MDR}})^2 + V(\chi_{\mathrm{MDR}};\alpha(\varepsilon)),$$

$$p_{\chi_{\mathrm{MDR}}} = \frac{1}{2} K_{\chi_{\mathrm{MDR}}} (\dot{\chi}_{\mathrm{MDR}})^2 - V(\chi_{\mathrm{MDR}};\alpha(\varepsilon)).$$

Local conservation $\nabla_\mu T^{\mu\nu} = 0$ leads to the continuity equation:

$$\dot{\rho}_{\chi_{\mathrm{MDR}}} + 3H \left(\rho_{\chi_{\mathrm{MDR}}} + p_{\chi_{\mathrm{MDR}}} \right) = 0.$$

Thus, the energy-conserving dynamics of the matter-dynamics rate χ_{MDR} holds.

The physical meaning is:

- The kinetic term
  $\frac{1}{2} K_{\chi_{\mathrm{MDR}}} (\dot{\chi}_{\mathrm{MDR}})^2$
  describes the local rate of matter dynamics.
- The potential term
  $V(\chi_{\mathrm{MDR}};\alpha(\varepsilon))$
  stores the epoch-dependent energy of space expansion.

Geometric Framework (Declaration)

ISOCH does not vary the geometry (no Einstein-Hilbert term). The FLRW background acts as a normalizing metric framework with $H(t) = \dot{a} / a$. Feedback (backreaction) on H does not occur through $\delta S / \delta g_{\mu\nu}$, but indirectly via the empirically determined energy density $\rho_{\chi_{\mathrm{MDR}}}$.

This separation is conceptual; the GR limit remains preserved:

$$\chi_{\mathrm{MDR}} \to 1
\; \Rightarrow \;
\rho_{\chi_{\mathrm{MDR}}} \to 0,
\; p_{\chi_{\mathrm{MDR}}} \to 0.$$

Normalization to the Critical Density

The epoch coordinate ε is canonically defined, following Part 2, as

$$\varepsilon \equiv \ln a
\; \Longleftrightarrow \;$$

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This definition does not alter any physical statement but fixes the normalization of the epoch coordinate within the energy-density analysis.

The cosmological energy balance is expressed through the ratio to the critical density ρ_c:

$$\rho_c = \frac{3 H_0^2}{8 \pi G}.$$

For the matter-dynamics rate (normalized to H_0), the following applies:

$$\frac{\rho_{\chi_{\mathrm{MDR}}}}{\rho_c}
= \frac{8 \pi G}{3 H_0^2}
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  \frac{1}{2} K_{\chi_{\mathrm{MDR}}} (\dot{\chi}_{\mathrm{MDR}})^2
  + V(\chi_{\mathrm{MDR}};\alpha(\varepsilon))
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If $K_{\chi_{\mathrm{MDR}}}$ is scaled to H_0^2, the dimensionless density component is obtained as:

$$\Omega_{\chi_{\mathrm{MDR}}}
= \frac{\rho_{\chi_{\mathrm{MDR}}}}{\rho_c}
= \frac{K_{\chi_{\mathrm{MDR}}}}{6 \pi G H_0^2} (\dot{\chi}_{\mathrm{MDR}})^2
 + \frac{8 \pi G}{3 H_0^2} V(\chi_{\mathrm{MDR}};\alpha(\varepsilon)).$$

The subsequent numerical calibration of the parameters

$$\frac{\Lambda_{\chi_{\mathrm{MDR}}}^3}{K_{\chi_{\mathrm{MDR}}}}
\quad \text{and} \quad
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is carried out exclusively on the basis of the previously defined theoretical potential structure.

Observational data are used solely for external scaling and do not enter as part of the variation

or the density equation. Consequently, the action and equation-of-motion system of the

χ_{MDR} dynamics remains unchanged, and the link between theory and empirical input

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From the calibration of the potential parameters, for the present epoch ($\varepsilon = 0$) one obtains:

$$\Omega_{\chi_{\mathrm{MDR}}} \approx 0.73,$$

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matter-dynamics rate χ_{MDR} thus assumes exactly the energetic role of dark energy,

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and thus

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The variational equation then reduces to the geometric GR case:

$$H^2 = \frac{8 \pi G}{3} \, \rho_m.$$

This corresponds exactly to the standard Friedmann equation of GR; ISOCH provides this expression without any additional approximation or auxiliary term. Thus, the ISOCH limit transition is unambiguous:

$$\lim_{\chi_{\mathrm{MDR}} \to 1} \left(\rho_{\chi_{\mathrm{MDR}}}, p_{\chi_{\mathrm{MDR}}} \right) = (0, 0) \; \Rightarrow \; \text{GR limit satisfied}.$$

The ISOCH extension therefore contains no additive Λ constant; the observed energy density arises entirely from the epochal dynamics of the matter-dynamics rate χ_{MDR}. The classical GR formulation is exactly recovered in the limit $\varepsilon \to 0$.

Quantitative Evolution of Energy Density

The epoch-dependent energy density follows from the potential profile. For the quadratic relaxation potential:

$$V(\chi_{\mathrm{MDR}};\alpha(\varepsilon)) = \frac{1}{2} m_{\chi_{\mathrm{MDR}}}^2 (\chi_{\mathrm{MDR}} - 1)^2,$$

it follows that

$$\rho_{\chi_{\mathrm{MDR}}}(\varepsilon)$$

$$
= \frac{1}{2} K_{\chi_{\mathrm{MDR}}} (\dot{\chi}_{\mathrm{MDR}})^2
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$$

Numerical integration shows an asymptotic decrease

$\rho_{\chi_{\mathrm{MDR}}}(\varepsilon) \propto e^{-3 H_0 t}$

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- The ISOCH energy density of the matter-dynamics rate dynamically replaces the Λ-term of GR.
- The coupling between expansion (H) and $\dot{\chi}_{\mathrm{MDR}}$ generates an epoch-dependent energy density that drifts toward a constant value in the late universe.
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Measurement Relation for Redshift

The observed redshift arises from the difference in material process rates between emission and observation:

$$
1 + z = \mathcal{M}\left(
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\right),
$$

where $\mathcal{M}$ is determined by ISOCH normalization; no photonic energy loss is postulated.

Conclusion

- The matter-dynamics rate χ_{MDR} provides a closed energy balance with well-defined density and pressure components.
- Normalization to ρ_c yields a present-day fraction of approximately 0.73, consistent with the observed dark energy.
- In the limit $\chi_{\mathrm{MDR}} \to 1$, ISOCH reduces exactly to GR; $\rho_{\chi_{\mathrm{MDR}}} \to 0$, $p_{\chi_{\mathrm{MDR}}} \to 0$.
- No independent Λ constant exists — the energy of spatial expansion arises from the epoch-dependent MDR dynamics itself.
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[END MACHINE-READABLE FORMULAS-PRIMARY SOURCE]